Electron-phonon kinetics

$$\left|M^{\pm q}_{k\to k'}\right|^2 = \delta_{k'-k,\mp q} \quad \frac{1}{L^3} B(q)$$

$$B(q) = B_0 q$$

$$B_0 = \hbar \frac{\Xi^2}{2\rho s}$$

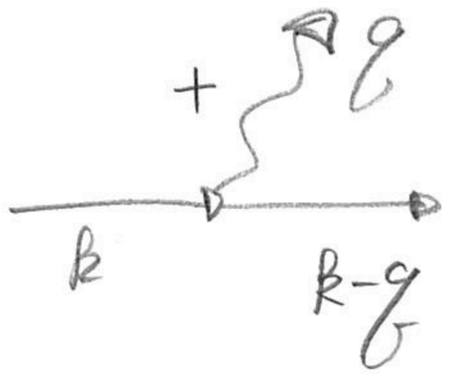
$$W^{\pm q}_{k\to k'} = \frac{2\pi}{\hbar}\left|M^{q}_{k\to k'}\right|^2 \left(N_q + \frac{1}{2} \pm \frac{1}{2}\right)$$

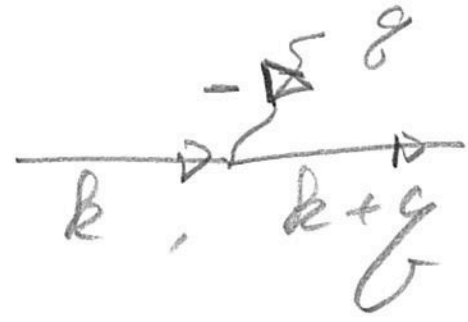
$$\cdot \delta(\epsilon_k - \epsilon_{k'} \mp \hbar\omega_q)$$

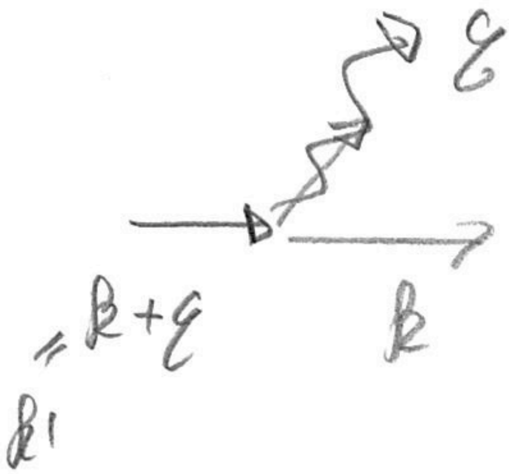
$$I_{e\text{-}ph}[f,N] = \sum_{k',q}\Big\{ W^{-,q}_{k\to k'}\, f_k(1-f_{k'})\, \cancel{N_q} +$$

$$+ W^{+,q}_{k\to k'}\, f_k(1-f_{k'})\, \cancel{(1+N_q)} +$$

$$+ W^{+,q}_{k'\to k}\, f_{k'}(1-f_k)\, \cancel{(N_q+1)} + W^{-,q}_{k',k}\, f_{k'}(1-f_k)\, \cancel{N_q}\Big\}$$


+
q
k
k-q


-
q
k
k+q


q
k'= k+q
k

$$= \sum_{k',q}{}' \frac{B_q}{L^3} \Big\{$$

$$- \delta_{k'-k,q}\, f_k (1 - f_{k'}) N_q\, \delta(\epsilon_k - \epsilon_{k'} + \omega_q) +$$

$$- \delta_{k'-k,-q}\, f_k (1 - f_{k'}) (1 + N_q)\, \delta(\epsilon_k - \epsilon_{k'} - \omega_q) +$$

$$+ \delta_{k-k',q}\, f_{k'} (1 - f_k) (1 + N_q)\, \delta(\epsilon_{k'} - \epsilon_k - \omega_q) +$$

$$+ \delta_{k-k',q}\, f_{k'} (1 - f_k) N_q\, \delta(\epsilon_{k'} - \epsilon_k + \omega_q) \Big\} =$$

$$= \int (d^d q)\, B_q \Big\{ - f_k (1 - f_{k+q}) N_q\, \delta(\epsilon_k - \epsilon_{k+q} + \omega_q) -$$

$$- f_k (1 - f_{k-q}) (1 + N_q)\, \delta(\epsilon_k - \epsilon_{k-q} - \omega_q)$$

$$+ f_{k+q} (1 - f_k) (1 + N_q)\, \delta(\epsilon_{k+q} - \epsilon_k - \omega_q) +$$

$$+ f_{k-q} (1 - f_k) N_q\, \delta(\epsilon_{k-q} - \epsilon_k + \omega_q) \Big\} =$$

$$= \int (d^d q)\, B_q \Big\{ \Big[\overbrace{-f_k(1-f_{k-q})(1+N_q)}^{\text{emmision of phonon "q"}} + \overbrace{f_{k-q}(1-f_k)N_q}^{\text{absorbtion of } q} \Big] \cdot$$

$$\cdot\, \delta(\epsilon_k - \epsilon_{k-q} - \omega_q) +$$

$$+ \Big[-f_k(1-f_{k+q})N_q + f_{k+q}(1-f_k)(1+N_q) \Big] \times$$

$$\times\, \delta(\epsilon_k - \epsilon_{k+q} + \omega_q)$$

Estimate characteristic life times

out going life time

$$\frac{1}{\tau_{out}(k)} = \int (d^d q)\, B(q) \left[1 - f_{k-q}\right](1+N_q)\, \delta(\epsilon_k - \epsilon_{k-q} - \omega_q)$$

$$\epsilon_k \cong v_F(k - k_F)$$

$$\omega_q = s q$$

$$\delta(v_F|k| - v_F|\vec{k} - \vec{q}| - s q) =$$

$$= \frac{1}{v_F}\, \delta\left(k - |\vec{k} - \vec{q}| - \frac{s}{v}\, q\right)$$

High temperatures

$T \gg \omega_D$

$$N(q) = \frac{1}{e^{\omega_q/T} - 1} \simeq \frac{T}{\omega_q}$$

$$s q \sim \omega_D$$

$$q \sim \omega_D/s \sim \frac{k_F s}{s} \sim k_F$$

$$\frac{1}{\tau_{out}} \simeq \int (d^d q) B'(q) \, (1 - f_{k-q}) \, T/\omega_q \, \delta(\epsilon_k - \epsilon_{k-q})$$

$$= T \int (d^d q) \frac{B'(q)}{\omega_q} (1 - f_{k-q}) \, \delta(\epsilon_k - \epsilon_{k-q})$$

elastic scattering

$$\frac{1}{\tau_{out}} \sim T$$

$$\sigma \propto \tau \propto T^{-1}$$

Low temperatures

$T \ll \omega_D$

energy conservation

$qs = |\epsilon_k - \epsilon_{k-q}| = \vec{v} \cdot \vec{q}$

perpendicular

$sq \sim T$

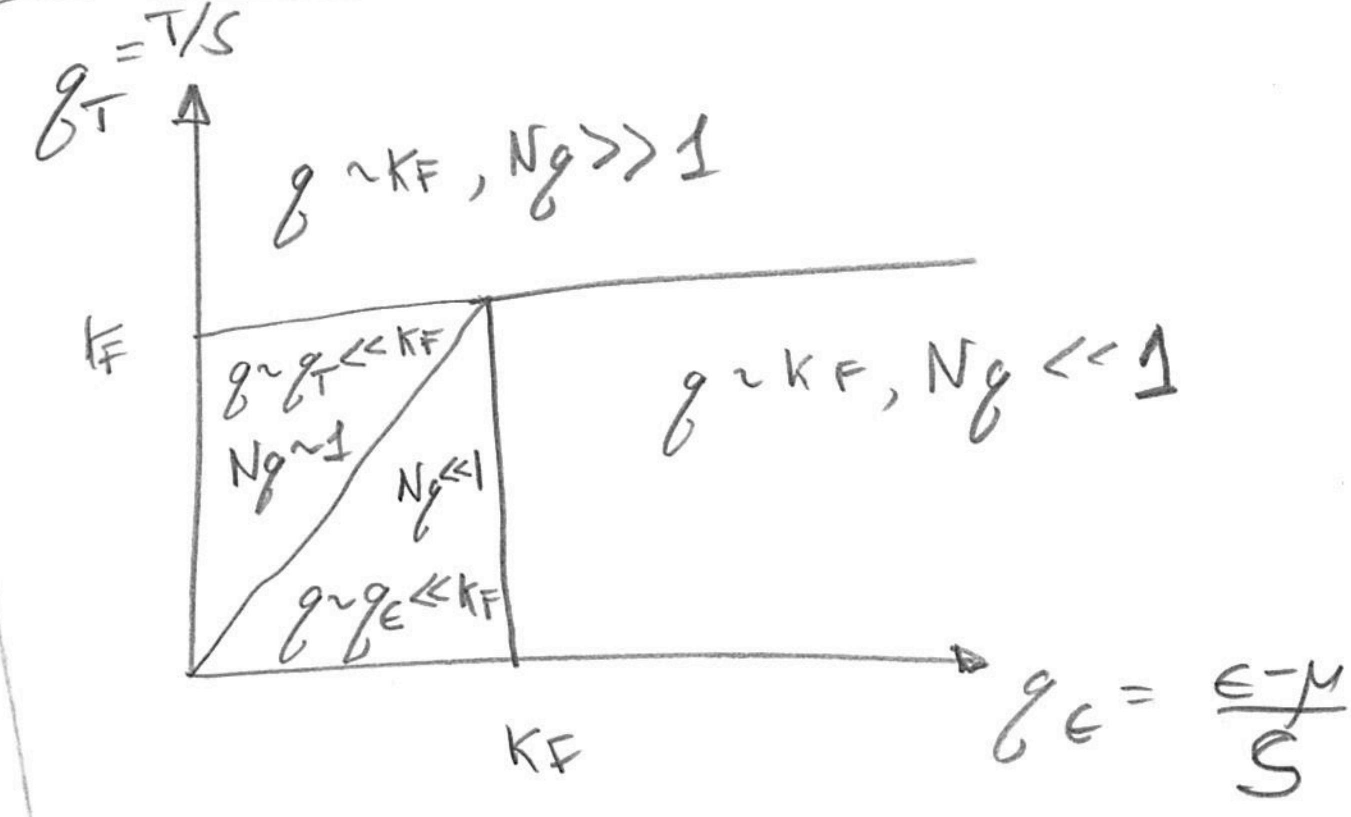
$\theta \sim q_T / k_F \ll 1$

$q \sim T/s \ll k_F$

Small change of the momentum

(small angle scattering)

$$\frac{1}{\tau_{out}(\epsilon)} = \frac{1}{\tau_F} \left(\frac{\epsilon - \mu}{k_F s} \right)^3$$



Electron-phonon scattering time

Boltzmann gas $T \gg \epsilon_F$

$$1/\tau_k = \sum_{k'} W_{k\to k'}$$

$$W^{\pm q}_{k\to k'} = \frac{2\pi}{h} |M^{\pm q}_{k\to k'}|^2 \left(N_q + \frac{1}{2} \pm \frac{1}{2}\right) \delta(\epsilon_k - \epsilon_{k'} \mp \omega_q),$$

Simple band $\epsilon_k = k^2/2m$

Scattering of thermal electrons

$\epsilon_k \sim T$

Consider the process with phonon absorption and emmision separately

$$1/\tau_k = 1/\tau_k^A + 1/\tau_k^E$$

$$1/\tau_k^A = \sum_{k',q} \frac{2\pi}{h} |M^{-q}_{k\to k'}|^2 N_q \, \delta(\epsilon_k - \epsilon_{k'} + \omega_q) =$$

$$= \frac{2\pi}{h} \int (d^d k')(d^d q) \, \delta_{k'-k,q} \, B(q) N_q \, \delta(\epsilon_k - \epsilon_{k'} + \omega_q)$$

$$= \frac{2\pi}{h} \int (d^d q) \, B(q) N(q) \, \delta(\epsilon_k - \epsilon_{q-k} + \omega_q)$$

$$= \frac{2\pi}{h} \int_0^{k_D} dq\, q^2 \int_0^{\pi} \sin\theta\, d\theta\, \frac{1}{(2\pi)^2}\, B(q)\, N(\omega_q)$$

$$\delta\left(\frac{k^2}{2m} - \frac{(\vec{k}-\vec{q})^2}{2m} + s q\right) =$$

$$= \frac{2\pi}{h} \frac{1}{(2\pi)^2} \int_0^{k_D} dq\, q^2 \int_0^{1} dx\, B(q)\, N(\omega_q)$$

$$\delta\left(\frac{kq}{m} x - \frac{q^2}{2m} + s q\right) =$$

$$= \frac{1}{2\pi h} \int_0^{k_D} dq\, q \int_0^{1} dx\, B(q)\, N(\omega_q)\, \delta\left(\frac{k}{m} x - \frac{q}{2m} + s\right)$$

$$= \frac{m}{2\pi h k} \int_0^{k_D} dq\, q\, B(q)\, N(\omega_q)$$

Adding em~~m~~ission we find

$$1/\tau(k) = \frac{m}{2\pi k} \int_0^{k_D} dq\, q\, B(q)\, \cancel{N(\omega_q)}\, (2N_q + 1)$$

we set $h = 1$

Deformation potential model

$$B(q) = B_0\, q$$

$$B_0 = \frac{\Xi^2}{2\rho s}$$

$$1/\tau(k) = \frac{m}{2\pi k} B_0 \int_0^{k_D} dq\, q^2 \left(2N(\omega_q)+1\right) =$$

$$= \frac{m}{2\pi k} B_0 \int_0^{sk_D/T} dx\, x^2 \left(2N(x)+1\right)\left(T/s\right)^3$$

If $\frac{sk_D}{T} \ll 1$, i.e. $T \gg \omega_D$ $\quad N(x) = \frac{1}{e^x-1} \simeq \frac{1}{x}$

$$1/\tau(k) = \frac{mB_0}{2\pi k}\left(\frac{T}{s}\right)^3 \int_0^{\frac{sk_D}{T}} dx\, x = \frac{mB_0 T^3}{2\pi s^3 k}\left(\frac{sk_D}{T}\right)^2 \frac{1}{2}$$

$$= \frac{mB_0}{4\pi s} \frac{k_D^2\, T}{k} = \frac{1}{4\pi} \frac{m\,\Xi^2}{2\rho s^2} \frac{k_D^2}{k} T$$

$$= \underbrace{\frac{1}{4\pi} \frac{\Xi^2 k_D^3}{2\rho s^2}}_{1/\tau_{DA}} \frac{mT}{k k_D} = \frac{1}{\tau_{DA}} \frac{mT}{k k_D}$$

$\leftarrow$ nominal scattering time

$$\left[\frac{mT}{k k_D}\right] = \left[\frac{m v^2 m}{k^2}\right] = \left[\frac{(mv)^2}{k^2}\right] = \left[\frac{k^2}{k^2}\right] = 1$$

Fermi-gas

$T \ll \epsilon_F$ (page 103 of G.L.)

$$1/\tau(k) = \sum_{k'} \Big[W_{k\to k'} (1 - f(k') + W_{k'\to k} f(k') \Big]$$

$$W^{\pm q}_{k\to k'} = \frac{2\pi}{h} |M^{\pm q}_{k\to k'}|^2 (N_q + \tfrac{1}{2} \pm \tfrac{1}{2}) \delta(\epsilon_k - \epsilon_{k'} \mp \hbar\omega_q)$$

Consider again the processes of emmission and absorbtion separately. Consider $A \leftrightarrow -$

$$1/\tau^A(k) = \frac{2\pi}{h}\int (dk')(dq)\, B(q) \Big\{ \delta(k'-k-q)\, N_q\, \delta(\epsilon_k - \epsilon_{k'} + \omega_q)(1-f(k')) + \delta(k-k'-q)\, N_q\, \delta(\epsilon_{k'} - \epsilon_k + \omega_q) f(k') \Big\} =$$

$$= \frac{2\pi}{h} \int (dq) B(q) \Big\{ N_q\, \delta(\epsilon_k - \epsilon_{k+q} + \omega_q)(1 - f(k+q)) + N_q\, \delta(\epsilon_{k-q} - \epsilon_k + \omega_q) f(k-q) \Big\} =$$

$$= \frac{2\pi}{h} \int (dq) B(q) N_{\omega_q} \Big\{ \delta(\epsilon_k - \epsilon_{k+q} + \omega_q)(1-f(k+q)) + \delta(\epsilon_{k-q} - \epsilon_k + \omega_q) f(k-q) \Big\}$$

$$= \frac{2\pi}{h} \int (dq) B(q) N_{\omega_q} \Big\{ \delta(\epsilon_k - \epsilon_{k+q} + \omega_q)(1 - f(\epsilon_k + \omega_q)) + \delta(\epsilon_k - \epsilon_{k-q} - \omega_q) f(\epsilon_k - \omega_q) \Big\}$$

detailed balance

$$f(k)\,(1-f(k'))\; W_{k\to k'} = f_{k'}(1-f_k)\; W_{k'\to k}$$

$$W_{k'\to k}\, f(k') = f(k)\,\frac{(1-f(k'))}{1-f(k)}\; W_{k\to k'}$$

$$1/\tau_k = \sum_{k'}{}' \left\{ W_{k\to k'}\,(1-f(k'))\left[1+\frac{f(k)}{1-f(k)}\right]\right\} =$$

$$= \sum_{k'} W_{k\to k'}\;\frac{1-f(k')}{1-f(k)}$$

Absorbtion process :

$$1/\tau_k^A = \frac{2\pi}{h}\int (dk')(dq)\, B(q)\,\delta(k'-k-q)\,\cdot$$

$$\cdot\,\delta(\epsilon_k-\epsilon_{k'}+\omega_q)\, N_{\omega_q}\,\frac{(1-f(\epsilon_{k'}))}{1-f(\epsilon_k)} =$$

$$= \frac{2\pi}{h}\int (d^3q)\, B(q)\,\delta(\epsilon_k-\epsilon_{k+q}+\omega_q)\,\times$$

$$N_{\omega_q}\,\frac{1-f(\epsilon_k+\omega_q)}{1-f(\epsilon_k)}$$

$$\epsilon_k - \epsilon_{k+q} + \omega_q =$$

$$= \frac{k^2}{2m} - \frac{(\vec{k}+\vec{q})^2}{2m} + sq =$$

$$= -\frac{\vec{k}\cdot\vec{q}}{m} - \frac{q^2}{2m} + sq \simeq$$

$$\simeq -v_F \cos\theta\, q + sq - q^2/2m$$

$$\delta(\epsilon_k - \epsilon_{k+q} + \omega_q) = \frac{1}{q}\,\delta(-v_F\cos\theta + s - q/2m) =$$

$$= \frac{1}{q v_F}\,\delta(-\cos\theta + s/v_F - q/2mv_F) =$$

$$= \frac{1}{q v_F}\,\delta\left(\cos\theta + \frac{q}{2k_F} - s/v\right)$$

compare $q/2k_F > s/v_F$

$$q > \frac{2k_F}{v_F} s = \frac{2m v_F s}{v} = ms$$

for $q > ms$ we can neglect s/v

$$\delta(\epsilon_k - \epsilon_{k+q} + \omega_q) \simeq \frac{1}{q v_F}\,\delta(\cos\theta + q/2k_F)$$

$$\int (d^3 q) = \frac{2\pi}{(2\pi)^3} \int_0^{k_D} q^2 dq \int_{-1}^{1} dx =$$

$$= \frac{1}{(2\pi)^2} \int_0^{k_D} q^2 dq \int_0^{1} dx$$

$$\int (d^3 q)\, \delta(\epsilon_k - \epsilon_{k+q} + \omega_q) = \frac{1}{(2\pi)^2} \frac{1}{v_F}$$

$$\int_0^{k_D} q^2 dq \; \frac{1}{q} \int_{-1}^{1} \delta(x + q/2k_F)\, dx =$$

$$= \frac{1}{(2\pi)^2 v_F} \int_0^{k_D} q\, dq$$

$$1/\tau_k^A = \frac{2\pi}{h} \frac{1}{(2\pi)^2 v_F} \int_0^{k_D} q\, dq \; B(q) N_{\omega_q} \frac{1 - f_{\epsilon_k + \omega}}{1 - f_{\epsilon_k}}$$

Similarily

$$1/\tau_k^E = \frac{2\pi}{h} \frac{1}{(2\pi)^2 v_F} \int_0^{k_D} q\, dq\, B(q)\, (1 + N_{\omega_q}) \frac{1 - f_{\epsilon_k - \omega}}{1 - f_{\epsilon_k}}$$

$$1/\tau_k = 1/\tau_k^A + 1/\tau_k^E$$

$$\frac{1}{\tau_k} = \frac{1}{2\pi v_F} \int_0^{k_D} dq\, q\, B_q \overbrace{\left[N_\omega \frac{1-f_{\epsilon+\omega_q}}{1-f_\epsilon} + (1+N_\omega)\frac{1-f_{\epsilon-}}{1-f_\epsilon} \right.}^{\Phi_+}$$

$T \gg \omega_D \qquad N_\omega \sim T/\omega_D$

$$\frac{1}{\tau_k} = \frac{1}{2\pi v_F} \int_0^{k_D} dq\, q\, B_q \frac{T}{\omega_D} \underbrace{\frac{2-f(\epsilon+\omega)-f(\epsilon-\omega)}{1-f_\epsilon}}_{\frac{2-2f(\epsilon)}{1-f(\epsilon)} \approx 1}$$

$$= \frac{1}{2\pi v_F} \int_0^{k_D} dq\, q\, B_q\, T/\omega_D =$$

$$= \frac{T}{2\pi v_F \omega_D} \int_0^{k_D} dq\, q\, B_q = \frac{T B_0}{2\pi v_F \omega_D} \int_0^{k_D} dq\, q^2 =$$

$$= \frac{1}{3} k_D^3 \frac{T B_0}{2\pi v_F \omega_D} = \frac{k_D^3\, T}{6\pi v_F\, k_D s} \frac{\Xi^2}{2\rho s} =$$

$$= \frac{k_D^2\, \Xi^2}{12\pi v_F \rho s^2} T \approx \frac{1}{\tau_{DA}} \frac{T}{v_F k_D}$$

Low temperatures, $T \ll \omega_D$

$$\Phi_+ = \psi^+ + \psi^- =$$

$$= (N_\omega + 1) \frac{1 - f(\epsilon - \omega)}{1 - f(\epsilon)} + N_\omega \frac{1 - f(\epsilon + \omega)}{1 - f(\epsilon)}$$

Above F.S. $\epsilon > 0$ $\quad N_\omega = 0,\ f(\epsilon) = 0$

$$\Phi_+ \simeq 1 - f(\epsilon - \omega)$$

$$1/\tau_k = \frac{1}{2\pi v_F} \int_0^{k_D} dq\, q\, B_q \left(1 - f(\epsilon - \omega)\right)$$

if $\epsilon > s k_D$ $\quad f(\epsilon - \omega) = 0$

$$1/\tau_k = \frac{1}{2\pi v_F} \int_0^{k_D} dq\, q\, B_q =$$

$$= \frac{1}{2\pi v_F} B_0 \int_0^{k_D} dq\, q^2 = \frac{B_0 k_D^3}{6\pi v_F} =$$

$$= \frac{k_D^3}{6\pi v_F} \frac{\Xi^2}{2\rho s} = \frac{k_D^3 \Xi^2}{12\pi v_F \rho s} \propto$$

$$= \frac{1}{\bar\tau_{DA}} \underbrace{s/v_F \ // \ \overline{\tau_F}}$$

$$\tau_k \propto \bar\tau_{DA} \frac{v_F}{s} \gg \tilde\tau_{DA}$$

Due to Pauli blocking

If $\epsilon \ll s k_D$

$\epsilon - \omega_q > 0$

$\epsilon > s q$

integration over $q < \epsilon/s$

$$1/\tau_R = \frac{1}{2\pi v_F} \int_0^{\epsilon/s} dq\, B_q \cdot q = \frac{B_0}{2\pi v_F} \int_0^{\epsilon/s} dq\, q^2 =$$

$$= \frac{B_0}{2\pi v_F} \, q^3/3 \Big|_0^{\epsilon/s} = \frac{2}{3} \frac{1}{\tau_F} \left(\frac{\epsilon}{2 k_D s} \right)^3$$

Note: for sinlicity we assume $k_D \simeq k_F$

ϵ is measured from the F.S.